Notre Dame University
Faculty of Natural and Applied Sciences
Department of Mathematics and Statistics

MAT 326

Probability & Statistics for Engineers
Exam #1

Friday November 21, 2008

Duration: 55 minutes

Name: ______________________

Section: T Th

Instructor: Dr. [illegible]

Grade: 80

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Please note that you have 5questions and 5pages

Round all your answers to 3 digits after the decimal point.

1) **(15 points)** From a group of 7 students of which 5 are Females, we are selecting randomly 3 students. Let X be a random variable to represent the number of females among the selected 3 students.

5F 2M 3 are selected

a) Find the probability distribution function $f(x)$ of X.

$P(X=1) = \frac{C_1^5 C_2^2}{C_3^7} = \frac{5}{35} = \frac{1}{7}$

$P(X=2) = \frac{C_2^5 C_1^2}{C_3^7} = \frac{10 \times 2}{35} = \frac{4}{7}$

$P(X=3) = \frac{C_3^5}{C_3^7} = \frac{10}{35} = \frac{2}{7}$

X	1	2	3
P(x)	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$

$$P(x) = \begin{cases} \frac{1}{7} & x = 1 \\ \frac{4}{7} & x = 2 \\ \frac{2}{7} & x = 3 \end{cases}$$

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$f(x) = P(X = x) = \{$

b) Find $E(X)$.

$E(x) = \sum x\, P(x) = 1 \times \frac{1}{7} + 2 \times \frac{4}{7} + 3 \times \frac{2}{7} = \frac{1}{7} + \frac{8}{7} + \frac{6}{7} = \frac{15}{7}$.

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c) Find $P(X \geq 2)$.

$P(X \geq 2) =$ ~~scribble~~ $P(X=2) + P(X=3)$

$= \frac{4}{7} + \frac{2}{7} = \frac{6}{7}$.

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2) **(16 points)** The probability that any one vehicle will turn left at a particular intersection is 0.2. The left-turn lane at this intersection has room for three vehicles.

a) If five vehicles arrive at this intersection while the light is red, find the probability that the left-turn lane will hold all the vehicles that want to turn left.

Let X be a R.V to represent the number of vehicules that want to turn left.

X: binomial with $n=5$, $p=0.2$

0

$p(X=5) = C^5_5 \, p^5 q^0 = (0.2)^5 = 3.2 \times 10^{-4}$.

b) Find the probability that six vehicles must arrive at the intersection while the light is red to fill up the left-turn lane.

Let Y be a R.V to represent the number of vehicules that must arrive to fill up the left turn:

Y: negative binomial with $n=6$, $p=0.2$.

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$P(Y_3=6) = C^5_2 (0.2)^3 (0.8)^3 = 0.04$.

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3) (24 points) The proportion of daily time that a small industrial robot is in use is a random variable X with probability density function

$$f(x)=\begin{cases}2(1-x) & \text{for } 0<x<1\\ 0 & \text{elsewhere.}\end{cases}$$

a) Find the cumulative distribution function $F(x)$ of X.

for $b<0$: $F(b)=\int_{-\infty}^{b} f(x)dx = 0$.

for $0\le b\le 1$: $F(b)=\int_{-\infty}^{b} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{b} 2(1-x)dx = 2\left(x-\frac{x^2}{2}\right)\Big|_0^b$

$= 2\left(b-\frac{b^2}{2}\right)$

for $b>1$: $F(b)=\int_{-\infty}^{0} f(x)dx + \int_{0}^{1} 2(1-x)dx + \int_{1}^{\infty} f(x)dx = 2\left(1-\frac{1}{2}\right)=1$

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$$F(x)=\begin{cases}0 & \text{for } x\le 0\\ 2\left(x-\frac{x^2}{2}\right) & 0\le x\le 1\\ 1 & x>1\end{cases}$$

b) For the robot under study, the daily profit $\$Y$ is given by $Y=180X-20$. Find $E(Y)$.

$E(X)=\int_0^1 x f(x)dx = \int_0^1 (2x-2x^2)dx = 2\int_0^1 (x-x^2)dx = 2\left(\frac{x^2}{2}-\frac{x^3}{3}\right)\Big|_0^1 = 2\left(\frac{1}{2}-\frac{1}{3}\right)$

$= 2\left(\frac{1}{6}\right)=\frac{1}{3}$.

$E(Y)=E(180X-20) = 180E(X)-20 = 180\times\frac{1}{3}-20 = 40.$

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c) Use $F(x)$ to find the probability that in a randomly selected day, the profit will be greater than or equal to \$70.

$P(Y\ge 70) = P(180X-20\ge 70) = P(180X\ge 90) = P(X\ge 0.5)$.

$= 1-F(0.5) = 1-2\left(0.5-\frac{(0.5)^2}{2}\right) = 0.25$.

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4) **(20 points)** The life length "in hundreds of hours" of certain type of fuses is a random variable X with probability density function given by

$$f(x)=\begin{cases}\frac{1}{3}e^{-x/3} & x>0\\ 0 & \text{elsewhere.}\end{cases}$$

$\beta = 300$

a) If one such fuse is randomly selected, what is the probability that its life length is less than 450 hours?

let X be a R.V representing the life length of the fuse
X: exponential with $\beta = 300$ h.

$P(X<450) = 1 - P(X>450) = 1 - e^{-\frac{450}{300}} = 1 - 0.223$
$= 0.777$

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b) An electronic system needs two such fuses, one in the primary system and the other one in the backup system which comes into use only if the primary fuse fails. Find the probability that total life length of these two fuses is greater than 600 hours.

independent let Y be a R.V to represent the life length of the two fuse.
$Y = X_1 + X_2$ $\Rightarrow$ Y is a gamma R.V with $\alpha = \alpha_1 + \alpha_2 = 2$.
$\alpha=1$ $\alpha=1$ $\beta = 300$.
$\beta=300$ $\beta=300$

$$f(x) = \begin{cases}\frac{1}{300^2} x e^{-\frac{x}{300}} & x>0\\ 0 & \text{elsewhere}\end{cases}$$

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$P(Y>600) = 1 - P(Y<600) = 1 - \int_0^{600} \frac{1}{300^2} x e^{-\frac{x}{300}} dx$

$1 - \frac{1}{300^2}\left[e^{-\frac{x}{300}}(-300x - 300^2)\right]_0^{600}$

$= 1 - \frac{1}{300}\left[-600e^{-\frac{600}{300}} - 300e^{-\frac{600}{300}}\right] = 1 - \frac{1}{300}\left[81.2 - 40.6\right]$

$= 1 - 0.135 = \boxed{0.865}$

$= 3e^{-2} = 0.406$

$-x e^{-\frac{x}{300}}$ $e^{-\frac{x}{300}}$
$-300 e^{-\frac{x}{300}}$
$300 e^{-\frac{x}{300}}$

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5) (25 points) A firm produces capacitors, their resistances are independent normally distributed with a mean of 90 mega ohms and a standard deviation of 10 mega ohms.

a) Find the probability that a randomly selected capacitor will have resistance higher than 100 mega ohms.

$\mu = 90$
$S = 10$

X: R.V representing the resistance of the capacitors
X normal with $\mu = 90$ and $S = 10$

$P(X > 100) = P(Z > \frac{100-90}{10}) = P(Z > 1) = 0.5 - A_1 = 0.5 - 0.343$
$= 0.157.$

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b) Two capacitors were randomly selected. Find the probability that both "each of them" will have resistance higher than 100 mega ohms.

let Y be a random variable representing the number of capacitors among the two selected to have a resistance higher than 100 MΩ.

Y: binomial with n = 2, p = 0.157 ⇒ q = 1 − 0.157 = 0.843.

$P(Y = 2) = C_2^2 \, p^2 q^0 = (0.157)^2 = 0.0246.$

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c) An electronic system needs 4 of these capacitors to be connected in series. Find the probability that their total resistance will be less than or equal 400 mega ohms.

$Y = X_1 + X_2 + X_3 + X_4$ independent ⇒ Y is normal with →

what is Y ?

$\mu_Y = 90 + 90 + 90 + 90 = 360.$

$S_Y = \sqrt{10^2 + 10^2 + 10^2 + 10^2} = 20$

$P(Y \leq 400) = P(Z \leq \frac{400-360}{20}) = P(Z \leq 2) = 0.5 + A_2$
$= 0.5 + 0.4772$
$= 0.9772.$

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